

# APPENDIX E

## Table of Constants and Formulas for Control Charts\*

Subgroup Size	$\bar{X}$ and R Charts				$\bar{X}$ and s Charts			
	Chart for Averages	Chart for Ranges (R)		Chart for Averages	Chart for Ranges (R)			
	Control Limits Factor	Divisors to Estimate $\sigma_X$	Factors for Control Limits		Control Limits Factor	Divisors to Estimate $\sigma_X$	Factors for Control Limits	
	$A_2$	$d_2$	$D_3$	$D_4$	$A_3$	$c_4$	$B_3$	$B_4$
2	1.880	1.128	—	3.267	2.659	0.7979	—	3.267
3	1.023	1.693	—	2.574	1.954	0.8862	—	2.568
4	0.729	2.059	—	2.282	1.628	0.9213	—	2.266
5	0.577	2.326	—	2.114	1.427	0.9400	—	2.089
6	0.483	2.534	—	2.004	1.287	0.9515	0.030	1.970
7	0.419	2.704	0.076	1.924	1.182	0.9594	0.118	1.882
8	0.373	2.847	0.136	1.864	1.099	0.9650	0.185	1.815
9	0.337	2.970	0.184	1.816	1.032	0.9693	0.239	1.761
10	0.308	3.078	0.223	1.777	0.975	0.9727	0.284	1.716
11	0.285	3.173	0.256	1.744	0.927	0.9754	0.321	1.679
12	0.266	3.258	0.283	1.717	0.886	0.9776	0.354	1.646
13	0.249	3.336	0.307	1.693	0.850	0.9794	0.382	1.618
14	0.235	3.407	0.328	1.672	0.817	0.9810	0.406	1.594
15	0.223	3.472	0.347	1.653	0.789	0.9823	0.428	1.572
16	0.212	3.532	0.363	1.637	0.763	0.9835	0.448	1.552
17	0.203	3.588	0.378	1.622	0.739	0.9845	0.466	1.534
18	0.194	3.640	0.391	1.608	0.718	0.9854	0.482	1.518
19	0.187	3.689	0.403	1.597	0.698	0.9862	0.497	1.503
20	0.180	3.735	0.415	1.585	0.680	0.9869	0.510	1.490
21	0.173	3.778	0.425	1.575	0.663	0.9876	0.523	1.477
22	0.167	3.819	0.434	1.566	0.647	0.9882	0.534	1.466
23	0.162	3.858	0.443	1.557	0.633	0.9887	0.545	1.455
24	0.157	3.895	0.451	1.548	0.619	0.9892	0.555	1.445
25	0.153	3.931	0.459	1.541	0.606	0.9896	0.565	1.435

	Centerline	Control Limits	
$\bar{X}$ and R Charts	$CL_{\bar{X}} = \bar{\bar{X}}$	$UCL_{\bar{X}} = \bar{\bar{X}} + A_2 \bar{R}$	$LCL_{\bar{X}} = \bar{\bar{X}} - A_2 \bar{R}$
	$CL_R = \bar{R}$	$UCL_R = D_4 \bar{R}$	$LCL_R = D_3 \bar{R}$
$\bar{X}$ and s Charts	$CL_{\bar{X}} = \bar{\bar{X}}$	$UCL_{\bar{X}} = \bar{\bar{X}} + A_3 \bar{S}$	$LCL_{\bar{X}} = \bar{\bar{X}} - A_3 \bar{S}$
	$CL_s = \bar{s}$	$UCL_s = B_4 \bar{s}$	$LCL_s = B_3 \bar{s}$

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**APPENDIX E - Table of Constants and Formulas for Control Charts (Cont.)**

Subgroup Size	Median Charts**				Charts for Individuals			
	Chart for Medians	Chart for Ranges (R)			Chart for Individuals	Chart for Ranges (R)		
	Control Limits Factor	Divisors to Estimate $\sigma_X$	Factors for Control Limits		Control Limits Factor	Divisors to Estimate $\sigma_X$	Factors for Control Limits	
	$\bar{A}_2$	$d_2$	$D_3$	$D_4$	$E_2$	$d_2$	$D_3$	$D_4$
2	1.880	1.128	—	3.267	2.660	1.128	—	3.267
3	1.187	1.693	—	2.574	1.772	1.693	—	2.574
4	0.796	2.059	—	2.282	1.457	2.059	—	2.282
5	0.691	2.326	—	2.114	1.290	2.326	—	2.114
6	0.548	2.534	—	2.004	1.184	2.534	—	2.004
7	0.508	2.704	0.076	1.924	1.109	2.704	0.076	1.924
8	0.433	2.847	0.136	1.864	1.054	2.847	0.136	1.864
9	0.412	2.970	0.184	1.816	1.010	2.970	0.184	1.816
10	0.362	3.078	0.223	1.777	0.975	3.078	0.223	1.777

	<b>Centerline</b>	<b>Control Limits</b>	
Median Charts	$CL_{\bar{X}} = \bar{X}$	$UCL_{\bar{X}} = \bar{X} + \bar{A}_2 \bar{R}$	$LCL_{\bar{X}} = \bar{X} - \bar{A}_2 \bar{R}$
	$CL_{\bar{R}} = \bar{R}$	$UCL_{\bar{R}} = D_4 \bar{R}$	$LCL_{\bar{R}} = D_3 \bar{R}$
Charts for Individuals	$CL_X = \bar{X}$	$UCL_X = \bar{X} + E_2 \bar{R}$	$LCL_X = \bar{X} - E_2 \bar{R}$
	$CL_R = \bar{R}$	$UCL_R = D_4 \bar{R}$	$LCL_R = D_3 \bar{R}$

| For extended  $d_2$  tables see the MSA Manual 3<sup>rd</sup> edition.

\*\*  $\bar{A}_2$  factors derived from ASTM-STP-15D Data and Efficiency Tables contained in Dixon and Massey (1969), page 488.

## APPENDIX E - Table of Constants and Formulas for Control Charts (Cont.)

### Attributes Charts

	Centerline	Control Limits	
<b>p chart</b> for proportions of units in a category	$CL_p = \bar{p}$	Samples not necessarily of constant size	
		$UCL_{p_i} = \bar{p} + 3 \frac{\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n_i}}$	$LCL_{p_i} = \bar{p} - 3 \frac{\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n_i}}$
		If the sample size is constant ( $n$ )	
		$UCL_p = \bar{p} + 3 \frac{\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n}}$	$LCL_p = \bar{p} - 3 \frac{\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n}}$
<b>np chart</b> for number/rate of units in a category	$CL_{np} = \overline{np}$	$UCL_{np} = \overline{np} + 3 \sqrt{\overline{np}(1-\frac{\overline{np}}{n})}$ $= \overline{np} + 3 \sqrt{\overline{np}(1-\bar{p})}$	$LCL_{np} = \overline{np} - 3 \sqrt{\overline{np}(1-\frac{\overline{np}}{n})}$ $= \overline{np} - 3 \sqrt{\overline{np}(1-\bar{p})}$
<b>c chart</b> for number of incidences in one or more categories	$CL_c = \bar{c}$	$UCL_c = \bar{c} + 3\sqrt{\bar{c}}$	$LCL_c = \bar{c} - 3\sqrt{\bar{c}}$
<b>u chart</b> for number of incidences per unit in one or more categories	$CL_u = \bar{u}$	Samples not necessarily of constant size	
		$UCL_{u_i} = \bar{u} + \frac{3\sqrt{\bar{u}}}{\sqrt{n_i}}$ $= \bar{u} + 3 \frac{\sqrt{\bar{u}}}{\sqrt{n_i}}$	$LCL_{u_i} = \bar{u} - \frac{3\sqrt{\bar{u}}}{\sqrt{n_i}}$ $= \bar{u} - 3 \frac{\sqrt{\bar{u}}}{\sqrt{n_i}}$
		Using average sample size	
		$UCL_u = \bar{u} + \frac{3\sqrt{\bar{u}}}{\sqrt{\bar{n}}}$ $= \bar{u} + 3 \frac{\sqrt{\bar{u}}}{\sqrt{\bar{n}}}$	$LCL_u = \bar{u} - \frac{3\sqrt{\bar{u}}}{\sqrt{\bar{n}}}$ $= \bar{u} - 3 \frac{\sqrt{\bar{u}}}{\sqrt{\bar{n}}}$
		If the sample size is constant ( $n$ )	
		$UCL_u = \bar{u} + \frac{3\sqrt{\bar{u}}}{\sqrt{n}}$ $= \bar{u} + 3 \frac{\sqrt{\bar{u}}}{\sqrt{n}}$	$LCL_u = \bar{u} - \frac{3\sqrt{\bar{u}}}{\sqrt{n}}$ $= \bar{u} - 3 \frac{\sqrt{\bar{u}}}{\sqrt{n}}$