

Selected Formulas

CHAPTER 2

Relative Frequency = (frequency)/ n

$$\bar{x} = \frac{\sum x}{n}$$

$$s^2 = \frac{\sum (x - \bar{x})^2}{n - 1} = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1}$$

$$s = \sqrt{s^2}$$

$$z = \frac{x - \mu}{\sigma} = \frac{x - \bar{x}}{s}$$

Chebyshev = At least $\left(1 - \frac{1}{k^2}\right)100\%$

$$\text{IQR} = Q_U - Q_L$$

CHAPTER 3

$$P(A^c) = 1 - P(A)$$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= P(A) + P(B) \text{ if } A \text{ and } B \text{ mutually exclusive} \end{aligned}$$

$$\begin{aligned} P(A \cap B) &= P(A|B) \cdot P(B) = P(B|A) \cdot P(A) \\ &= P(A) \cdot P(B) \text{ if } A \text{ and } B \text{ independent} \end{aligned}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\binom{N}{n} = \frac{N!}{n!(N - n)!}$$

Bayes's: $P(S_i|A) =$

$$\frac{P(S_i)P(A|S_i)}{P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + \cdots + P(S_k)P(A|S_k)}$$

CHAPTER 6

CI for μ : $\bar{x} \pm (z_{\alpha/2})\sigma/\sqrt{n}$ (large n)
 $\bar{x} \pm (t_{\alpha/2})s/\sqrt{n}$ (small n , σ unknown)

$$\text{CI for } p: \hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$\text{Estimating } \mu: n = (z_{\alpha/2})^2(\sigma^2)/(\text{SE})^2$$

$$\text{Estimating } p: n = (z_{\alpha/2})^2(pq)/(\text{SE})^2$$

$$\text{CI for } \sigma^2: = \frac{(n - 1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n - 1)s^2}{\chi^2_{1 - \alpha/2}}$$

CHAPTER 4

Key Formulas

Random Variable	Prob. Dist'n	Mean	Variance
General Discrete:	Table, formula, or graph for $p(x)$	$\sum_{\text{all } x} x \cdot p(x)$	$\sum_{\text{all } x} (x - \mu)^2 \cdot p(x)$
Binomial:	$p(x) = \binom{n}{x} p^x q^{n-x}$ $x = 0, 1, 2, \dots, n$	np	npq
Poisson:	$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ $x = 0, 1, 2, \dots$	λ	λ
Uniform:	$f(x) = 1/(d - c)$ $(c \leq x \leq d)$	$(c + d)/2$	$(d - c)^2/12$
Normal:	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2[(x-\mu)/\sigma]^2}$	μ	σ^2
Standard Normal:	$f(z) = \frac{1}{\sqrt{2\pi}} e^{-1/2(z)^2}$ $z = (x - \mu)/\sigma$	$\mu = 0$	$\sigma^2 = 1$

CHAPTER 5

Sample Mean (large n): Approx. normal, $\mu_{\bar{x}} = \mu$ $\sigma_{\bar{x}}^2 = \sigma^2/n$

CHAPTER 7

$$\text{Test for } \mu: z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \text{ (large } n\text{)}$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \text{ (small } n, \sigma \text{ unknown)}$$

$$\text{Test for } p: z = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0/n}}$$

$$\text{Test for } \sigma^2: \chi^2 = (n - 1)s^2/(\sigma_0)^2$$

CHAPTER 8

CI for $\mu_1 - \mu_2$:

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \left\{ \text{(large } n_1 \text{ and } n_2) \right\}$$

Test for $\mu_1 - \mu_2$:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \left\{ \text{(large } n_1 \text{ and } n_2) \right\}$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

CI for $\mu_1 - \mu_2$:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} \left\{ \text{(small } n_1 \text{ and/or } n_2) \right\}$$

Test for $\mu_1 - \mu_2$:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \left\{ \text{(small } n_1 \text{ and/or } n_2) \right\}$$

$$\text{CI for } \mu_d: \bar{x}_d \pm t_{\alpha/2} \frac{s_d}{\sqrt{n}}$$

$$\text{Test for } \mu_d: t = \frac{\bar{x}_d - \mu_d}{s_d/\sqrt{n}}$$

$$\text{CI for } p_1 - p_2: (\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

$$\text{Test for } p_1 - p_2: z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

$$\text{Test for } (\sigma_1^2/\sigma_2^2): F = (s_1^2/s_2^2)$$

$$\text{Estimating } \mu_1 - \mu_2: n_1 = n_2 = (z_{\alpha/2})^2(\sigma_1^2 + \sigma_2^2)/(\text{ME})^2$$

$$\text{Estimating } p_1 - p_2: n_1 = n_2 = (z_{\alpha/2})^2(p_1 q_1 + p_2 q_2)/(\text{ME})^2$$

CHAPTER 9

ANOVA Test for completely randomized design:

$$F = \text{MST/MSE}$$

ANOVA Test for randomized block design:

$$F = \text{MST/MSE}$$

ANOVA Test for factorial design interaction:

$$F = \text{MS}(A \times B)/\text{MSE}$$

Pairwise comparisons: $c = k(k - 1)/2$

CHAPTER 10

$$\text{Multinomial test: } \chi^2 = \sum \frac{(n_i - E_i)^2}{E_i}$$

$$E_i = n(p_{i0})$$

$$\text{Contingency table test: } \chi^2 = \sum \frac{(n_{ij} - E_{ij})^2}{E_{ij}}$$

$$E_{ij} = \frac{R_i C_j}{n}$$

CHAPTER 11

$$SS_{xx} = \sum (x - \bar{x})^2 = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$SS_{yy} = \sum (y - \bar{y})^2 = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$SS_{xy} = \sum (x - \bar{x})(y - \bar{y}) = \sum xy - \frac{(\sum x)(\sum y)}{n}$$

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}}$$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}} \sqrt{SS_{yy}}}$$

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CHAPTER 11 (cont'd)

$$s^2 = \frac{SSE}{n - 2}$$

$$s = \sqrt{s^2}$$

$$r^2 = \frac{SS_{yy} - SSE}{SS_{yy}}$$

$$\text{CI for } \beta_1: \hat{\beta}_1 \pm (t_{\alpha/2})s/\sqrt{SS_{xx}}$$

$$\text{Test for } \beta_1: t = \frac{\hat{\beta}_1 - 0}{s/\sqrt{SS_{xx}}}$$

$$\text{CI for } E(y) \text{ when } x = x_p: \hat{y} \pm t_{\alpha/2}s\sqrt{\frac{1}{n} + \frac{(x_p - \bar{x})^2}{SS_{xx}}}$$

$$\text{CI for } y \text{ when } x = x_p: \hat{y} \pm t_{\alpha/2}s\sqrt{1 + \frac{1}{n} + \frac{(x_p - \bar{x})^2}{SS_{xx}}}$$

Quadratic Model (QN x):

$$E(y) = \beta_0 + \beta_1x + \beta_2x^2$$

Complete 2nd-Order Model (QN x 's):

$$E(y) = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_1x_2 + \beta_4x_1^2 + \beta_5x_2^2$$

Dummy Variable Model (QL x):

$$E(y) = \beta_0 + \beta_1x_1 + \beta_2x_2$$

where $x_1 = \{1 \text{ if A, } 0 \text{ if not}\}$, $x_2 = \{1 \text{ if B, } 0 \text{ if not}\}$

$$MSE = s^2 = \frac{SSE}{n - (k + 1)}$$

$$R^2 = \frac{SS_{yy} - SSE}{SS_{yy}}$$

$$R_a^2 = 1 - \left[\frac{(n - 1)}{n - (k + 1)} \right] (1 - R^2)$$

$$\text{Test for overall model: } F = \frac{MS(\text{Model})}{MSE}$$

$$\text{Test for individual } \beta: t = \frac{\hat{\beta}_i - 0}{s_{\hat{\beta}_i}}$$

$$\text{CI for } \beta_i: \hat{\beta}_i \pm (t_{\alpha/2})s_{\hat{\beta}_i}$$

$$\text{Nested model } F \text{ test: } F = \frac{(SSE_R - SSE_C)/\# \beta\text{'s tested}}{MSE_C}$$

CHAPTER 12

First-Order Model (QN x 's):

$$E(y) = \beta_0 + \beta_1x_1 + \beta_2x_2 + \dots + \beta_kx_k$$

Interaction Model (QN x 's):

$$E(y) = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_1x_2$$

CHAPTER 13

Key Formulas

Control Chart	Centerline	Control Limits	A–B Boundary	B–C Boundary
$\bar{x}$ -chart	$\bar{\bar{x}} = \frac{\sum_{i=1}^k \bar{x}_i}{k}$	$\bar{\bar{x}} \pm A_2\bar{R}$ or $\bar{\bar{x}} \pm 2\frac{(\bar{R}/d_2)}{\sqrt{n}}$	$\bar{\bar{x}} \pm \frac{2}{3}(A_2\bar{R})$ or $\bar{\bar{x}} \pm 2\frac{(\bar{R}/d_2)}{\sqrt{n}}$	$\bar{\bar{x}} \pm \frac{1}{3}(A_2\bar{R})$
R -chart	$\bar{R} = \frac{\sum_{i=1}^k R_i}{k}$	$(\bar{R}D_3, \bar{R}D_4)$	$\bar{R} \pm 2d_3\left(\frac{\bar{R}}{d_2}\right)$	$\bar{R} \pm d_3\left(\frac{\bar{R}}{d_2}\right)$
p -chart	$\bar{p} = \frac{\text{Total number defectives}}{\text{Total number units sampled}}$	$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$	$\bar{p} \pm 2\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$	$\bar{p} \pm \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$
Capability index: $C_p = (\text{USL} - \text{LSL})/6\sigma$				